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THE FREE ENERGY OF HOT QED AT THREE AND A HALF LOOPS *

Rajesh R. Parwani†
Service de Physique Theorique, CE-Saclay
F-91191, Gif-sur-Yvette, France

and

Claudio Corianò‡
High Energy Physics Division, Argonne National Laboratory
9700 South Cass, Il 60439, USA.

Abstract

The computation of order e^4 and e^5 contributions to the pressure of massless quantum electrodynamics at a temperature T is overviewed.

The thermodynamic properties of a QED plasma may be determined from the partition function which can be obtained perturbatively using techniques borrowed from field theory at zero temperature (T). Indeed, the simplest approach to calculating the partition function, or the free energy, is to use the imaginary-time formalism whereby the Feynman rules are as at $T = 0$ but the energies take on discrete Matsubara values.

However a naive application of these $T = 0$ like Feynman rules soon leads to the appearance of power-like infrared (IR) singularities in diagrams. When these IR singularities from an infinite set of diagrams are resummed, one obtains an expansion in $\sqrt{e^2}$ rather than e^2, where e is the QED coupling. The best-known example of this phenomenon is the e^3 plasmon correction to the free energy of QED first found by Gell-Mann and Brueckner [1] in the nonrelativistic context and later calculated relativistically [2]. Physically, the appearance of these IR singularities and the consequent breakdown of the naive perturbative expansion is due to the Debye screening of electric fields in a plasma. Unlike at $T = 0$, the particles in the plasma are not free (modulo ultraviolet renormalisations) but perpertually under the influence of the surrounding particles.

In the language of field theory, the Debye screening manifests itself by the non-vanishing limit $\Pi_{00}(p_0 = 0, \vec{p} \to 0)$ of the electric polarization operator. To lowest order in the coupling, and in the limit of high temperature, the electric screening mass-squared is given by a one-loop calculation : $m^2 = \Pi^1(p_0 = 0, \vec{p} \to 0) = e^2T^2/3$.

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†E-mail: parwani@wasa.saclay.cea.fr

‡E-mail: coriano@hep.anl.gov

The presence of the large scale T means that the "loop correction" need not be small compared to the bare propagator. In imaginary time, this is the case for the zero mode (Matsubara frequency) of the electric propagator at small three momentum ($|\vec{p}| \sim eT$). (By contrast, static magnetic fields are unscreened in a QED plasma.)

In general there are three ways that one can account for large corrections, such as Debye screening, in order to restore the perturbative expansion :

(i) Continue with bare Feynman rules and resum by hand dangerous subsets of diagrams. This procedure is possible in simple cases but has the disadvantage that one must carefully identify the relevant diagrams, account for symmetry factors and prevent overcounting; or

(ii) Begin with the non-perturbative skeleton-expansion and truncate down. This is safe from the point of view of symmetry factors and overcounting but is practical only in simple cases; or

(iii) Re-organise the bare Lagrangian by adding and subtracting the dominant non-negligible effects (termed "hard thermal loops" by Braaten and Pisarski [3]). This method has the virtue that one continues with Feynman perturbation theory but with new effective propagators and vertices.

Usually, methods (i) and (ii) are computationally efficient only for static (zero external energy) Greens functions for which all the power counting analysis can be done in imaginary-time. For a non-static Greens function one first requires its physical definition in real time (either by analytic continuation from imaginary time or through the real-time formalism) and then the method advocated in Ref.[3] is probably the most efficient.

Recently we computed the order e^4 (3-loop) contribution to the free energy density of massless QED at temperature T, going beyond the e^3 term known for many years [2]. The method used was (i) with dimensional regularisation being extensively employed to regulate various singularities appearing at intermediate stages of the calculation. From the technical point of view the fourth order calculation required the evaluation of some complicated overlapping three-loop integrals that did not appear in a similar three-loop calculation in ϕ^4 theory [4].

Following the order e^4 calculation in Ref.[5], the e^5 ($3\frac{1}{2}$ loop) result was also obtained [6]. This latter term may be viewed as a correction to the three-loop result as a consequence of Debye screening, just as the e^3 term is a similar correction to the two-loop (order e^2) result. Computationally, the order e^3 calculation is *simpler* than the order e^2 calculation because only the zero mode of the photon is involved and the loop integral along that line becomes three-dimensional. Similarly, the e^5 piece was easier to obtain than the e^4 piece, the sum of complicated diagrams factorizing themselves into a product of simple one-loop integrals. The e^5 contribution was also reconsidered from the point of view of method (ii) in Ref.[7].

Remarkably, because the odd terms (e^3, e^5) are simple to calculate, one is able to write a general identity [6]. Consider the gauge-invariant contribution to the pressure of *massless* QED, at temperature T, at order $e^{2n}, (n \geq 1)$, coming from diagrams with one-fermion loop. Call this contribution P_{2n}^{1F}. Then the order e^{2n+3} piece is obtained by dressing the photons of P_{2n}^{1F} and is given by

$$P_{2n+3}^{1F} = \frac{e^3 T^2 N^{1/2}}{8\pi\sqrt{3}} \frac{\partial^2 P_{2n}^{1F}}{\partial \mu^2}|_{\mu=0} \quad , n \geq 1 \, . \tag{1}$$

On the right-hand-side of Ref.(1), μ is the chemical potential and N is the number of massless electron flavours. The generalisation of Eq.(1) to massive electrons at non-zero chemical potential is given in Ref.[7].

Thus at least in this one case the "IR problem" of perturbation theory at non-zero temperature has turned out to be a bonus in giving us a cute relation like Eq.(1). Unfortunately the relation is not quite useful since, as yet, we do not have a simple painless algorithm to get the even pieces.

Note added in proof : At time of writing (August 1994), the three-loop free-energy of hot Yang-Mills theory has been calculated [8].

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Appendix

The fine-structure constant at temperature T is $\alpha(T) = e^2(T)/4\pi$. Defining $g^2 = \alpha(T)N/\pi$, the pressure of QED with N massless Dirac fermions at nonzero temperature, T, is then given by :

$$\frac{P}{T^4} = a_0 + g^2 a_2 + g^3 a_3 + g^4(a_4 + b_4/N) + g^5(a_5 + b_5/N) + O(g^6)\,, \quad \text{(A.1)}$$

with

$$a_0 = \frac{\pi^2}{45}\,(1 + \frac{7}{4}N)\,, \quad \text{(A.2)}$$

$$a_2 = -\frac{5\pi^2}{72}\,, \quad \text{(A.3)}$$

$$a_3 = \frac{2\pi^2}{9\sqrt{3}}\,, \quad \text{(A.4)}$$

$$a_4 = -0.757 \pm 0.004\,, \quad \text{(A.5)}$$

$$b_4 = 0.658 \pm 0.006\,, \quad \text{(A.6)}$$

$$a_5 = \frac{\pi^2[1 - \gamma - \ln(4/\pi)]}{9\sqrt{3}} = 0.11473...\,, \quad \text{(A.7)}$$

$$b_5 = \frac{-\pi^2}{2\sqrt{3}}\,. \quad \text{(A.8)}$$